Code No: **R231105** **R23** **SET - 1**

I B. Tech I Semester Supplementary Examinations, June/July-2024
LINEAR ALGEBRA AND CALCULUS
(Common to All Branches)

Time: 3 hours Max. Marks: 70

*Note: 1. Question paper consists of two parts (**Part-A** and **Part-B**)*
*2. All the questions in **Part-A** is Compulsory*
*3. Answer **ONE** Question from **Each Unit** in **Part-B***

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PART –A (20 Marks)

1. a) Find the rank of the matrix $A=\begin{bmatrix}1 & 1 & -1 & 1\\ 1 & -1 & 2 & -1\\ 3 & 1 & 0 & 1\end{bmatrix}$. [2M]

b) Determine k such that the system of homogeneous equations $2x+y+z=0$, $x+y+3z=0$, $4x+3y+kz=0$ has non-trivial solution. [2M]

c) Let λ be an Eigen value of A and X be its corresponding eigenvector. Then, Prove that A^2 has Eigen value λ^2 and the corresponding eigenvector is X. [2M]

d) Identify the nature the quadratic form: $17x_1^2-30x_1x_2+17x_2^2$. [2M]

e) State Cauchy's mean value theorem. [2M]

f) State why Rolle's theorem cannot be applied to the function $f(x)=|x|$ in the interval $[-1,1]$. [2M]

g) Find the first order partial derivatives of $u(x,y)=\log(x^2+y^2)$. [2M]

h) If $u=x\sin y$, $v=y\sin x$, find $\frac{\partial(u,v)}{\partial(x,y)}$. [2M]

i) Evaluate $\int_2^4\int_1^2 \frac{dydx}{xy}$. [2M]

j) Evaluate $\int_0^c\int_0^b\int_0^a (x^2+y^2+z^2)\,dxdydz$. [2M]

PART – B (50 MARKS)

UNIT-I

2. a) Solve the following system of linear equations by using Gauss elimination method $2x+y-z=4$, $x-y+2z=-2$, $-x+2y-z=2$. [5M]

b) Using Gauss-Jordan method, find the inverse of the matrix $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 1 & 3 & 4\end{bmatrix}$. [5M]

(OR)

3 a) Solve the homogeneous system $3x+2y+z=0$, $2x+3z=0$, $y+5z=0$, $x+2y+3z=0$. [5M]

b) Perform four iterations of the Gauss-Jacobi iteration method for solving the system of equations $\begin{bmatrix}3 & 2 & 0\\ 2 & 3 & -1\\ 0 & -1 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}5\\ 4\\ 1\end{bmatrix}$. Take the initial approximation as $X^{(0)}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$. [5M]
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UNIT-II

4. a) Find the Eigen values and Eigen vectors of $A=\begin{bmatrix}1&0&0\\0&2&1\\2&0&3\end{bmatrix}$. [5M]

b) Verify Cayley-Hamilton theorem for $A=\begin{pmatrix}1&4\\2&3\end{pmatrix}$ and hence find A^{-1}. [5M]

(OR)

5. Reduce the quadratic form $x_1^2+5x_2^2+x_3^2+2x_1x_2+2x_2x_3+6x_3x_1$ into canonical form by orthogonal transformation. State matrix for transformation and determine the index, signature and nature of the quadratic form. [10M]

UNIT-III

6. a) Verify Rolle's theorem for $f(x)=(x-a)^m(x-b)^n$, where m, n are positive integers in $[a,b]$. [5M]

b) Verify Cauchy's mean value theorem for the functions $f(x)=\log_e x$ and $g(x)=\frac{1}{x}$ in the interval $[1,e]$. [5M]

(OR)

7. a) Verify Lagrange's Mean Value theorem for $f(x)=x(x-1)(x-2)$, $in\ [0,\frac{1}{2}]$. [5M]

b) Verify Maclaurin's theorem for $f(x)=(1-x)^{5/2}$ with Lagrange's form of remainder up to 3 terms when $x=1$. [5M]

UNIT-IV

8. a) If $u=\frac{x+y}{1-xy}$ and $v=\tan^{-1}x+\tan^{-1}y$, find $\frac{\partial(u,v)}{\partial(x,y)}$. Check whether u and v functionally related? If so, find this relationship. [5M]

b) Discuss the maxima and minima of $f(x,y)=x^4+y^4-2x^2+4xy-2y^2$. [5M]

(OR)

9. a) If $u=x^2+y^2+z^2$ and $x=e^{2t}$; $y=e^{2t}\cos 3t$; $z=e^{2t}\sin 3t$. Find $\frac{du}{dt}$ as a total derivative and verify the result by direct substitution. [5M]

b) Find the Taylor's expansion of $f(x,y)=e^x\cos y$ about the point $x=1$, $y=\frac{\pi}{4}$. [5M]

UNIT-V

10. a) Find the area bounded by the curves $y^2=4ax$ and $x^2=4ay$. [5M]

b) Evaluate $I=\int_0^a\int_0^x\int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$. [5M]

(OR)

11. a) By changing the order of integration, and hence evaluate $\int_0^1\int_{x^2}^{2-x} xy\ dy\ dx$. [5M]

b) Using Triple Integral, Find the volume of the sphere $x^2+y^2+z^2=a^2$. [5M]

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Code No: **R201101** **R20** **SET - 1**

I B. Tech I Semester Supplementary Examinations, June/July-2024

MATHEMATICS-I

(Common to All Branches)

Time: 3 hours Max. Marks: 70

Answer any FIVE Questions ONE Question from Each Unit
All Questions Carry Equal Marks

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UNIT – I

1 a) Test the Convergence of the series $\sum_{i=1}^{\infty} \frac{1.2.3\ldots n}{3.5.7\ldots(2n+1)}$ [7M]

b) Test the Convergence of the series $\sum_{i=1}^{\infty} (-1)^n \frac{1}{nlogn}$ [7M]

(OR)

2 a) Verify Lagrange's Rolle's value theorem for f(x) = $x^3 - x^2 - 5x+3$ 0n [0,4] [7M]

b) Obtain Taylor's series Expansion of sin x about x = π/4 [7M]

UNIT - II

3 a) Solve the Differential Equation $\left(y^4+2y\right) dx + \left(xy^3+2y^4-4x\right) dy = 0$ [7M]

b) The initial value problem governing the current i flowing in series R, L Circuit when voltage v(t) = t is applied is given by $Ri + L\frac{di}{dt} = t\ ,\quad t \geq 0$. Find the current i (t) at time t. [7M]

(OR)

4 a) Solve the Differential Equation $\frac{dy}{dx} + x(x+y) = x^3(x+y)^3 - 1$ [7M]

b) Determine the charge and current at any time in t in R-C circuit with R = 10 ohms, c= 2 Farad and E = 100 volts given that q(0) = 0. [7M]

UNIT - III

5 a) Solve the Differential Equation $\left(D^2-1\right) y = x \sin x$ [7M]

b) Solve the Differential Equation $(D^2 + 1)y = \cot x$ using method of variation of parameters [7M]

(OR)

6 a) Solve the Differential Equation $\left(x^2D^2+4xD+2\right)y = e^x$ [7M]

b) Solve the Differential Equation $y^{111} + y = \cos(2x-1)$ [7M]
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UNIT – IV

7 a) Verify Euler's theorem for the function $u = \sin^{-1}\left(\dfrac{x^3 + y^3 + z^3}{ax + by + cz}\right)$ [7M]

b) if $u = \dfrac{yz}{x}, v = \dfrac{zx}{y}, w = \dfrac{xy}{z}$ then find $J\left(\dfrac{x, y, z}{u, v, w}\right)$ [7M]

(OR)

8 a) if $u = xf\left(\dfrac{y}{x}\right) + g\left(\dfrac{y}{x}\right)$ then prove that $x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} = 0$ [7M]

b) Find the extreme values of $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$ [7M]

UNIT - V

9 a) Evaluate $\iiint \dfrac{dx\, dy\, dz}{x^2 + y^2 + z^2}$, taken over the volume bounded by the sphere $x^2 + y^2 + z^2 = a^2$ using spherical coordinate system. [7M]

b) Evaluate by change of order of integration $\int_0^1 \int_{e^x}^{e} \dfrac{1}{\log y} \, dydx$ [7M]

(OR)

10 a) Find the area which is inside the circle $r = 2a \cos \theta$ and outside the circle $r = a$. [7M]

b) Find the volume of the region common to the cone $\phi = b$ and the sphere $r = a$. [7M]

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Code No: **R19BS1101** **R19** **SET - 1**

I B. Tech I Semester Supplementary Examinations, June/July-2024
MATHEMATICS-I

(Common to CE, EEE, ME, ECE, CSE, Chem E, EIE, IT, Auto E, Min E, Pet E, Agri E)

Time: 3 hours Max. Marks: 75

Answer any FIVE Questions ONE Question from Each Unit
All Questions Carry Equal Marks

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UNIT-I

1. a) Test for convergence of the series (i) $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+...........$ [8M]

   (ii) $\sum_{n=1}^{\infty}\frac{n!\,2^n}{n^n}$

   b) State first mean value theorem and using it prove that $(0<a<b<1)$ $\frac{b-a}{1+b^2}<\tan^{-1}b-\tan^{-1}a<\frac{b-a}{1+a^2}$. Hence show that $\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\frac{4}{3}<\frac{\pi}{4}+\frac{1}{6}$. [7M]

(OR)

2. a) Test for convergence of the series $\sum_{n=1}^{\infty}\left(\frac{n+2}{n+3}\right)^n x^n$. [8M]

   b) Verify Cauchy's mean value theorem for the functions $f(x)=\log_e x$ and $g(x)=\frac{1}{x}$ in the interval $[1,e]$. [7M]

UNIT-II

3. a) Solve the differential equation $\frac{dy}{dx}=\frac{1}{(1+x^2)}\left(e^{\tan^{-1}x}-y\right)$. [8M]

   b) In a certain culture of bacteria the rate of increase is proportional to the number present. (i) If it is found that the number doubles in 4 hours, how many may be expected at the end of 12 hours. (ii) If there are 10^4 at the end of 3 hours and 4×10^4 at the end of 5 hours, how many were in the beginning. [7M]

(OR)

4. a) Solve the differential equation $\frac{dy}{dx}-y=y^2(\sin x+\cos x)$. [8M]

   b) Show that the family of confocal conics $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$ is "self-orthogonal". Here *a* and *b* are given constants. [7M]

UNIT-III

5. a) Solve $(D^2-4D+4)y=8(e^{2x}+\sin 2x+x^2)$. [8M]

   b) Solve the differential equation $\frac{d^2y}{dx^2}+4y=4\sec^2 2x$ by method of variation of parameters. [7M]

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(OR)

6. a) Solve $(D^2+1)y = e^{-x} + \cos x + x^3 + e^x \sin x$. [8M]

b) A circuit has in series an electromotive force given by $E = 200\cos 100\,t$ Volts, a resistor of 5 ohms, an inductor of 0.05 henrys, and a capacitor of 4×10^{-4} farads. If the initial current and the initial charge on the capacitor are both zero, find the charge on the capacitor at any time t > 0. [7M]

UNIT-IV

7. a) If $u(x,y) = \sin^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \tan u$. [8M]

b) If $u = x^2+y^2+z^2$ and $x = e^{2t}$; $y = e^{2t}\cos 3t$; $z = e^{2t}\sin 3t$. Find $\dfrac{du}{dt}$ as a total derivative. [7M]

(OR)

8. a) If $u = \dfrac{x+y}{1-xy}$ and $v = \tan^{-1}x + \tan^{-1}y$ find $\dfrac{\partial(u,v)}{\partial(x,y)}$. Are u and v functionally related If so, find their relationship. [8M]

b) Find the maxima and minima of $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. [7M]

UNIT-V

9. a) Evaluate $\iint_R x^2\,dx\,dy$, where R is the region in the first quadrant bounded by the lines $y = x,\ y = 0,\ x = 8$ and the curve $xy = 16$. [8M]

b) Find the area bounded by the curves $y^2 = 4ax$ and $x^2 = 4ay$. [7M]

(OR)

10 a) By changing the order of integration, and hence evaluate $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$. [8M]

b) Evaluate $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z\,dz\,dx\,dy$. [7M]

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Code No: **R161102** **R16** **SET - 1**

I B. Tech I Semester Supplementary Examinations, June/July-2024
MATHEMATICS-I

Time: 3 hours Max. Marks: 70

*Note: 1. Question paper consists of two parts (**Part-A** and **Part-B**)*
*2. All the questions in **Part-A** is Compulsory*
*3. Answer any **FOUR** Questions from **Part-B***

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PART –A (14 Marks)

1. a) Solve the differential equation $x\frac{dy}{dx} + y = 2x$ [2M]

b) Determine orthogonal trajectories of the family of parabolas $y^2 = 4ax$, where 'a' is the parameter. [2M]

c) Write Laplace transform of $e^{3t} + sinh4t + t^3 - 1$ [2M]

d) Show that $L\{\delta(t-a)\} = e^{-as}$ [2M]

e) Find $J = \frac{\partial(u,v)}{\partial(x,y)}$ if $u = x^2 - y^2$, $v = 2xy$ [2M]

f) From the partial differential equation by eliminating arbitrary constants from $z = ax + by + a^2 + b^2$ [2M]

g) Classify the Partial differential equation $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$ [2M]

PART –B (56 Marks)

2. a) $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x.\cos y + x} = 0$ Is the equation exact ? If so find the solution. [7M]

b) A body is originally at 80^0C and cools down to 60^0C in 20 minutes. If the temperature of the air is 40^0C, find the temperature of the body after 40 minutes. [7M]

3. a) Find solution of $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = x + e^{2x} + cos2x$ [7M]

b) Use method of variation of Parameters and find solution of $\frac{d^2y}{dx^2} + y = tan\, x$ [7M]

4. a) With the help of Convolution theorem and find $L^{-1}\left\{\frac{s}{(s^2+9)(s^2+16)}\right\}$ [7M]

b) By using Laplace transform method find solution of the differential equation $y'' + 7y' + 10y = 4e^{-3t}, y(0) = 0, y'(0) = -1.$ [7M]

5. a) Expand $f(x, y) = e^x log(1 + y)$ about the origin up to second degree terms using Taylor's theorem. [7M]

b) Determine the dimensions of a rectangular box requiring least material for its construction whose top is open is to have volume of 32 cubic feet. [7M]
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6. a) Find solution of the partial differential equation $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$. [7M]

   b) Find solution of the partial differential equation $p^2+pq=z^2$. [7M]

7. a) Solve the partial differential equation $\left(D^2 - 7DD^1 + 12D^{1^2}\right)z = e^{x-y}$ Where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$ [7M]

   b) Solve the partial differential equation $\left(D^3 - 4D^2D^1 + 4DD^{1^2}\right)z = 2\sin(3x+2y)$ Where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$ [7M]

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