

I B. Tech II Semester Regular Examinations, June/July - 2024
DIFFERENTIAL EQUATIONS AND VECTOR CALCULUS

(Common to all Branches)

Time: 3 hours

Max. Marks: 70

Note: 1. Question paper consists of two parts (Part-A and Part-B)

2. All the questions in Part-A is Compulsory

3. Answer ONE Question from each Unit in Part-B

PART -A (20 Marks)

1. a) Find the order of the D.E $\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = 5y$ [2M]
- b) Find the I.F of $\frac{dy}{dx} + \frac{y}{x} = x$ [2M]
- c) Find the P.I of $(D^2 + 2D + 3)y = -30$ [2M]
- d) Solve the differential equation $(D^2 + 5D + 6)y = 0$ [2M]
- e) Find the degree of partial differential equation $\left(\frac{\partial^2 z}{\partial x^2}\right)^3 - \frac{\partial^2 z}{\partial y^2} = e^x \sin y$ [2M]
- f) Solve the partial differential equation $(D^2 - a^2 D^1)z = 0$ [2M]
- g) Find the unit normal vector to the surface $\phi = xy + yz + zx - 3 = 0$ at (1,1,1) [2M]
- h) Find $\nabla\phi$ where $\phi = x^2y + yz^2 + zx - 2 = 0$ at (1,-1,0) [2M]
- i) Define line integral. [2M]
- j) Write the relation between the volume integral in terms of surface integral. [2M]

PART - B (50 MARKS)

UNIT-I

2. a) Solve the differential equation $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$ [5M]
- b) Solve the differential equation $\frac{dy}{dx} + \frac{x^2+y^2+x}{xy} = 0$ [5M]

(OR)

3. a) The growth rate of bacteria population is proportional to its size. Initially the population is 10,000, after 5 days it is 20,000. What is the population after 15 days. [5M]
- b) When a resistance R ohms is connected in series with an inductance L Henries an e.m.f of E volts i.e., $E = 10 \sin t$ then the current in the circuit if $i(0) = 0$. [5M]

UNIT-II

4. a) Solve the differential equation $(D^2 - 5D + 6)y = \sin 3x$ [5M]
- b) Solve the differential equation $(D^3 - 3D^2 - 10D + 24)y = x + 3$ [5M]

(OR)

5. a) Solve the differential equation $(D^2 + 4)y = \tan 2x$ using method of variation parameters. [5M]
- b) Solve the differential equation $(D^2 + 1)y = x \sin x$. [5M]



UNIT-III

6. a) Find partial differential equation by eliminating arbitrary function from $z = f(x - y)$. [5M]
 b) Solve the partial differential equation $p + 3q = 5z + \tan(y - 3x)$. [5M]

(OR)

7. a) Find partial differential equation by eliminating arbitrary constants from $\log(az - 1) = x + ay + b$ [5M]
 b) Solve the partial differential equation $(D^2 - D^2)z = \cos x \cos 2y$ [5M]

UNIT-IV

8. a) Prove that $\nabla(r^n) = nr^{n-2}\vec{r}$ [5M]
 b) Prove that $\nabla \times (\vec{A} \times \vec{B}) = (\nabla \cdot \vec{B})\vec{A} - (\nabla \cdot \vec{A})\vec{B} + (\vec{B} \cdot \nabla)\vec{A} - (\vec{A} \cdot \nabla)\vec{B}$ [5M]

(OR)

9. a) Find the directional derivative of $\phi = xy + yz + zx$ at A in the directional of \vec{AB} where $A(1, 2, -1)$ and $B(1, 2, 3)$ [5M]
 b) Show that $\vec{f} = \vec{a} \times \vec{r}$ is solenoidal where $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$ and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ [5M]

UNIT-V

10. Verify Green's theorem in plane for $\oint_C (x^2 - xy^3)dx + (y^2 - 2xy)dy$ where C is the square with $(0, 0)$, $(0, 2)$, $(2, 2)$ and $(2, 0)$ [10M]

(OR)

11. a) Prove that $\oint_C (f \nabla g) \cdot d\vec{r} = \int_S (\nabla f \times \nabla g) \cdot \vec{n} ds$ using stokes theorem. [5M]
 b) Evaluate $\int_S \vec{F} \cdot \vec{n} ds$ where $\vec{F} = z\vec{i} + x\vec{j} - 3y^2z\vec{k}$ and S is the surface $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$. [5M]



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- Note: 1. Question paper consists of two parts (Part-A and Part-B)*
2. All the questions in Part-A is Compulsory
3. Answer ONE Question from each Unit in Part-B

PART -A (20 Marks)

1. a) Find the degree of the differential equation $\left[5 + \left(\frac{dy}{dx}\right)^2\right]^{5/3} = \frac{2d^2y}{dx^2}$ [2M]
- b) Solve the differential equation $\frac{dy}{dx} = e^{-y}(e^x + x^2)$ [2M]
- c) Find the P.I of $(D^2 - 2D + 6)y = 3$ [2M]
- d) Solve the differential equation $(D^2 + 4)y = 0$ [2M]
- e) Find the order of partial differential equation $\left(\frac{\partial z}{\partial x}\right)^4 + \left(\frac{\partial^3 z}{\partial y^3}\right)^2 - \frac{\partial^3 z}{\partial y^2 \partial x} + z = \cos(x + y)$ [2M]
- f) Solve the partial differential equation $(D^2 - 4D^1)z = 0$ [2M]
- g) Find the unit normal vector to the surface $\phi = xy - z^2 = 0$ at $(4,1,2)$ [2M]
- h) Find $\text{div} \vec{f}$ where $\vec{f} = xi + 2yzj + xzk$ [2M]
- i) Define line integral. [2M]
- j) Write Green's theorem. [2M]

PART - B (50 MARKS)

UNIT-I

2. a) Solve the differential equation $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$ [5M]
- b) Solve the differential equation $y dx - x dy + (1+x^2)dx + x^2 \sin y dy = 0$ [5M]

(OR)

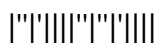
3. a) A bacterial culture, growing exponentially, increases from 100 to 400 gms in 10 hrs. [5M]
How much was present after 3 hrs. from the initial instant
- b) A body is originally at 80°C and cools down to 60°C in 20 minutes. If the temperature [5M]
of the air is 40°C , find the temperature of the body after 40 minutes

UNIT-II

4. a) Solve the differential equation $(D^2 + 4)y = \sec 2x$ using method of variation [5M]
parameters
- b) Solve the differential equation $(D^2 - 1)y = x^4$ [5M]

(OR)

5. a) Solve the differential equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 8e^{3x} \sin 2x$ [5M]
- b) Solve the differential equation $\frac{d^2y}{dx^2} + 4y = \sin 3x + \cos 2x$ [5M]



UNIT-III

6. a) Solve the partial differential equation $y^2p - xyq = x(z - 2y)$ [5M]
 b) Find partial differential equation by eliminating arbitrary function from $z = f(\sin x - \cos y)$ [5M]

(OR)

7. a) Solve the partial differential equation $(D^2 + DD^1 - 6D^{1^2})z = \cos(2x + y)$ [5M]
 b) Find partial differential equation by eliminating arbitrary constants from $x^2 + (y - b)^2 + (z - c)^2 = 64$ [5M]

UNIT-IV

8. a) Prove that $\nabla(\log r) = \frac{\vec{r}}{r^2}$ [5M]
 b) If \vec{f} , ϕ be differentiable vector and scalar functions respectively then prove that $\nabla \times (\phi \vec{f}) = (\nabla \phi) \times \vec{f} + \phi(\nabla \times \vec{f})$ [5M]

(OR)

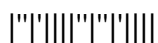
9. a) Find the directional derivative of $\phi = x^2yz + 4xz^2$ in the directional of $2\vec{i} - \vec{j} - 2\vec{k}$ at $(1, -2, -1)$ [5M]
 b) Prove that $r^n \vec{r}$ is solenoidal if $n = -3$ [5M]

UNIT-V

10. a) Apply Gauss Divergence theorem to compute $\iint_S \vec{F} \cdot \vec{n} ds$ where $\vec{F} = x\vec{i} - y\vec{j} + z\vec{k}$ over the surface of the cylinder $x^2 + y^2 = a^2$ bounded by the planes $z = 0, z = b$. [5M]
 b) Find the work done in moving particle in the force field $\vec{F} = 2x^2\vec{i} + (2yz - x)\vec{j} - 4z\vec{k}$ Along the straight Line $(0, 0, 0)$ to $(3, 1, 2)$. [5M]

(OR)

11. If $\vec{f} = (3x^2 - 2z)\vec{i} - 4xy\vec{j} - 5x\vec{k}$ Evaluate $\int_V \text{Cur } \vec{F} dv$, where v is volume [10M] bounded by the planes $x = 0; y = 0; z = 0$ and $3x + 2y - 3z = 6$



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 2. All the questions in Part-A is Compulsory
 3. Answer ONE Question from each Unit in Part-B*

PART -A (20 Marks)

1. a) Find the order of the differential equation $\left(\frac{dy}{dx}\right)^2 - 4y = 0$ [2M]
- b) Solve the differential equation $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$ [2M]
- c) Find the P.I of $(D^2 + 4)y = -5$ [2M]
- d) Solve the differential equation $(D^2 - 9)y = 0$ [2M]
- e) Find the degree of partial differential equation $\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$ [2M]
- f) Solve the partial differential equation $(D^2 + 9D^{12})z = 0$ [2M]
- g) Find the unit normal vector to the surface $\phi = x^2 + y^2 + z^2 - 6 = 0$ at (2,1,1) [2M]
- h) Find $\text{curl } \vec{f}$ where $\vec{f} = xi + yj + zk$ [2M]
- i) Define surface integral. [2M]
- j) Write Gauss divergence theorem. [2M]

PART - B (50 MARKS)

UNIT-I

2. a) Solve the differential equation $(1+x^2)\frac{dy}{dx} - 2xy = 2x(1+x^2); y(0)=1$ [5M]
- b) Solve the differential equation $(xe^{xy} + 2y)\frac{dy}{dx} + ye^{-xy} = 0$ [5M]

(OR)

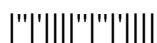
3. a) A body kept in air with temperature 25°C cools from 140°C to 80°C in 20 minutes. Find when the body cools down to 35°C . [5M]
- b) Find the current in electrical circuit is given by $L\frac{di}{dt} + iR = E$ where $E = 6$ volts, $R = 100$ ohms, $L = 0.1$ Henry. [5M]

UNIT-II

4. a) Solve the differential equation $y^{11} + 4y^1 + 5y = -2 \cosh x$ [5M]
- b) Solve the differential equation $(D^2 - 4D + 3)y = e^x \cos 2x$ [5M]

(OR)

5. a) Solve the differential equation $(D^3 + D^2 - D - 1)y = \cos 2x$ [5M]
- b) Solve the differential equation $(D^2 + 4)y = \cot 2x$ using method of variation parameters. [5M]



UNIT-III

6. a) Solve the partial differential equation $(4D^2 + 12DD^1 + 9D^{1^2})z = e^{3x-2y}$ [5M]
 b) Find partial differential equation by eliminating arbitrary constants from $x^2 + y^2 + (z - c)^2 = a^2$ [5M]

(OR)

7. a) Find partial differential equation by eliminating arbitrary function from $z = f(x + at) + g(x - at)$ [5M]
 b) Solve the partial differential equation $px + qy = 1$. [5M]

UNIT-IV

8. a) Prove that $\nabla \left(\frac{1}{r} \right) = \frac{-\vec{r}}{r^3}$ [5M]
 b) Prove that $div(\vec{A} \times \vec{B}) = \vec{B} \cdot curl \vec{A} - \vec{A} \cdot curl \vec{B}$ [5M]

(OR)

9. a) Find the directional derivative of $\phi = xyz$ in the directional of $\vec{i} + \vec{j} + \vec{k}$ at $(1, 2, 3)$ [5M]
 b) Show that $r^n \vec{r}$ is irrotational. [5M]

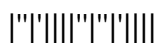
UNIT-V

10. a) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = 2x^2yz \vec{i} + x^2y \vec{j}$ where C is the curve $x = t, y = t^2, z = t^3$ from $t = 0$ to 1 [5M]
 b) Evaluate $\iint_S \vec{A} \cdot \vec{n} ds$ where $\vec{A} = yz \vec{i} + zx \vec{j} + xy \vec{k}$ and s is the part of sphere $x^2 + y^2 + z^2 = 9$ which lies in the first octant. [5M]

(OR)

11. Verify stokes theorem for $\vec{F} = x^2\vec{i} + xy\vec{j}$ around the square in $z = 0$ plane whose sides are along the lines $x = 0; y = 0; x = 1, y = 1$. [10M]

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- Note: 1. Question paper consists of two parts (Part-A and Part-B)*
2. All the questions in Part-A is Compulsory
3. Answer ONE Question from each Unit in Part-B

PART -A (20 Marks)

1. a) Find the degree of the differential equation $\left(\frac{dy}{dx}\right)^2 + 4y = 0$ [2M]
- b) Find the I.F of $\frac{dy}{dx} - \frac{y}{x} = x$ [2M]
- c) Find the P.I of $(D^2 + 5D + 6)y = 2$ [2M]
- d) Solve the differential equation $(D^2 + D + 1)y = 0$ [2M]
- e) Solve the partial differential equation $(D^2 + DD - 6D^2)z = 0$ [2M]
- f) Find the order of partial differential equation $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = z$ [2M]
- g) Find the unit normal vector to the surface $\phi = xy^2z - 2 = 0$ at (2,1,1). [2M]
- h) Define solenoidal vector. [2M]
- i) State stokes theorem. [2M]
- j) Define flux. [2M]

PART - B (50 MARKS)

UNIT-I

2. a) Solve the differential equation $x \frac{dy}{dx} + y = x^3 y^6$ [5M]
- b) Solve the differential equation $(x^2 + y^2)dx - 2xy dy = 0$ [5M]

(OR)

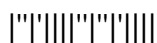
3. a) A resistance of 100 ohms an inductance of 0.5 Henry is connected in series with a battery of 20 volts. Find the current in the circuit as a function of t, if initially there is no current in the circuit. [5M]
- b) If the temperature of the air is 30°C and substance cools from 100°C to 70°C in 15 minutes, find when the temperature will be 40°C. [5M]

UNIT-II

4. a) Solve the differential equation $y^{11} - 4y^1 + 3y = 4e^{3x}$, $y(0) = -1$, $y'(0) = 3$ [5M]
- b) Solve the differential equation $(D^2 - 4D + 4)y = x^3 e^{2x}$ [5M]

(OR)

5. a) Solve the differential equation $(D^2 + 5D + 4)y = x^2 + 7x + 9$ [5M]
- b) Solve the differential equation $(D^2 + 4)y = x \cos 2x$ [5M]



UNIT-III

6. a) Find partial differential equation by eliminating arbitrary function from $z = f(2x + 3y) + g(3x - y)$ [5M]
 b) Solve the partial differential equation $yzp + zxq = xy$. [5M]

(OR)

7. a) Solve the partial differential equation $(D^2 + 2DD^1 + D^{1^2})z = x^2 + xy + y^2$ [5M]
 b) Find partial differential equation by eliminating arbitrary constants {l,m,n} from $lx + my + nz = a$ [5M]

UNIT-IV

8. a) Prove that $\nabla(e^{r^2}) = 2e^{r^2}\vec{r}$ [5M]
 b) If \vec{f} , ϕ be differentiable vector and scalar functions respectively then prove that $div(\phi\vec{f}) = (grad\phi) \cdot \vec{f} + \phi div\vec{f}$ [5M]

(OR)

9. a) In what direction from the point $(1, -2, -1)$ the directional derivative of $\phi = x^2yz + 4xz^2$ is maximum? What is the magnitude of this maximum [5M]
 b) Show that the vector $(x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ is irrotational and find its scalar potential. [5M]

UNIT-V

10. a) Evaluate $\oint_C \cos y dx + x(1 - \sin y)dy$ over a closed curve c is given by $x^2 + y^2 = 1; z = 0$. Using Green's theorem. [5M]
 b) If $\vec{F} = (x - y)\vec{i} + (x + y)\vec{j}$ Evaluate $\oint_C \vec{F} \cdot d\vec{r}$ around the curve $y = x^2$ and $y^2 = x$ [5M]

(OR)

11. Verify the Divergence theorem for $\vec{F} = 4xy\vec{i} - y^2\vec{j} + xz\vec{k}$ over the cube [10M] bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$.



I B. Tech II Semester Supplementary Examinations, June/July-2024**APPLIED PHYSICS**

(Common to EEE, ECE, EIE, ECT, CSE-AI&ML, CSE-AI, CSE-DS, CSE-AI&DS, AI&DS)

Time: 3 hours

Max. Marks: 70

*Answer any FIVE Questions ONE Question from Each Unit
All Questions Carry Equal Marks*

UNIT - I

1. a) i. Explain how colors are formed on thin oil films. [10M]
ii. Differentiate ordinary and extra-ordinary rays.
b) Calculate the minimum thickness of a soap bubble that results in constructive interference in the reflected light if the film is illuminated with a light of wavelength 600 nm. [4M]

(OR)

2. a) i. Obtain an expression for the dispersive power of a grating [10M]
ii. Define quarter wave and half wave plates.
b) The refractive index of calcite for ordinary ray is 1.658 and for extraordinary ray it is 1.486. The slice having a thickness of 0.9×10^{-4} cm is cut from the crystal. For what wavelength the slice acts as a half wave plate? [4M]

UNIT - II

3. a) What are the characteristics of a laser? Explain population inversion. [7M]
b) Write in brief about spontaneous emission, stimulated emission and pumping mechanism. [7M]

(OR)

4. a) Define acceptance angle and obtain the relation for it. [7M]
b) Explain the classification of optical fibres. Differentiate step index and graded index optical fibres. [7M]

UNIT - III

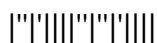
5. a) What is a matter wave? Explain the significance and properties of wave function. [7M]
b) Derive Schrodinger's time independent wave equation. [7M]

(OR)

6. a) What are the assumptions of quantum free electron theory? Obtain an expression for electrical conductivity. [10M]
b) Explain the classification of crystalline solids. [4M]

UNIT - IV

7. a) Derive Clausius –Mossotti equation. [10M]
b) Calculate the electronic polarizability of argon atom. Given dielectric constant is 1.0024 at NTP and $N = 2.7 \times 10^{25}$ atoms/m³. [4M]



(OR)

8. a) i. Discuss the origin of magnetic moment. [10M]
ii. Differentiate soft and hard magnetic materials with examples.
- b) When a magnetic material is subjected to a magnetic field of intensity 250 A/m, its relative permeability is 15. Calculate its magnetization and magnetic flux density. [4M]

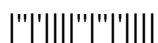
UNIT – V

9. a) State and explain Hall effect. What are its applications? [10M]
- b) Find the resistivity of intrinsic germanium at 300⁰K. Given that intrinsic density carriers is $2.5 \times 10^{19} /\text{m}^3$. [4M]

(OR)

10. a) i. Define critical magnetic field and flux quantization. [10M]
ii. Explain BCS theory of superconductivity.
- b) A superconducting material has an isotopic mass of 199.5 a.m.u. and critical temperature is 5K. Then calculate isotopic mass at 5.1k. [4M]

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I B. Tech II Semester Supplementary Examinations, June/July- 2024**MATHEMATICS-III**

(Common to CE, EEE, ECE, CSE, Chem E, EIE, IT, Auto E, Min E, Pet E)

Time: 3 hours

Max. Marks: 75

*Answer any FIVE Questions ONE Question from Each Unit
All Questions Carry Equal Marks*

UNIT-I

1. a) Find the directional derivative of the function $e^{2x} \cos yz$ at the origin in the direction to the tangent to the curve $x = a \sin t, y = a \cos t, z = at$ at $t = \frac{\pi}{4}$ [8M]

- b) Evaluate $\oint_C \sin y \, dx + (1 + \cos y) \, dy$ over the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; z = 0$ using Green's theorem [7M]

(OR)

2. a) Show that $\vec{f} = r^n (\vec{a} \times \vec{r})$ is solenoidal where $\vec{a} = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k}$ and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ [8M]

- b) Apply Gauss Divergence theorem to compute $\iint_S \vec{F} \cdot \vec{n} \, ds$ where $\vec{F} = x\vec{i} - y\vec{j} + z\vec{k}$ over the surface of the cylinder $x^2 + y^2 = a^2$ bounded by the planes. $z = 0, z = b$. [7M]

UNIT-II

3. a) Evaluate $\int_0^\infty e^{-t} \frac{\sin^2 t}{t} \, dt$ using Laplace transform [8M]

- b) Solve the D.E $y'' + y = \sin 3t, y(0) = y'(0) = 0$ using Laplace transform method [7M]

(OR)

4. a) Find the Laplace transform of $(t) = \begin{cases} \cos t, & 0 < t \leq \pi \\ \sin t, & t > \pi \end{cases}$ [8M]

- b) Find $L^{-1} \left\{ \frac{s-5}{(s^2-10s+29)^2} \right\}$ [7M]

UNIT-III

5. a) Find the Half range cosine series of $f(x) = x^2$ in $[0, \pi]$ [8M]

- b) Find the Fourier Sine transforms of $f(x) = \frac{e^{-ax}}{x}$ and hence deduce that [7M]

$$\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} \sin px \, dx = \tan^{-1} \left(\frac{p}{a} \right) - \tan^{-1} \left(\frac{p}{b} \right)$$



(OR)

6. a) Find the Fourier series of $f(x) = \begin{cases} \pi x & 0 < x < 1 \\ 0 & 1 < x < 2 \end{cases}$ [8M]
- b) Find Fourier transform of $f(x) = e^{-\frac{x^2}{2}}$, $-\infty < x < \infty$. [7M]

UNIT-IV

7. a) Form the partial differential equation of the following by eliminating arbitrary function from $z = y f(x^2 + z^2)$ [8M]
- b) Solve the partial differential equation $y^2 p - xyq = x(z - 2y)$ [7M]

(OR)

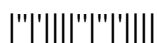
8. a) Solve the partial differential equation $yzp + xq = xy$ [8M]
- b) Solve the partial differential equation $z^2 = 1 + p^2 + q^2$ [7M]

UNIT-V

9. A rod of 30 cm long has its ends A and B kept at 20° C and 80° C respectively until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0° C and kept so. Find the resulting temperature function $u(x, t)$ taking $x=0$ at A. [15M]

(OR)

10. a) Solve the partial differential equation $(D^3 - 4D^2D^1 + 3DD^1^2)z = \sin(3x + 2y)$ [8M]
- b) Solve the partial differential equation $\frac{\partial u}{\partial x} + 4\frac{\partial u}{\partial y} = 0$, $u(0, y) = 3e^{-y}$ by variable separable method. [7M]



I B. Tech II Semester Supplementary Examinations, June/July-2024

MATHEMATICS-III

(Common to AE, AME, CE, CSE, IT, EIE, EEE, ME, ECE, Metal E, Min E, E Com E, Agri E, Chem E, PCE, PE)

Time: 3 hours

Max. Marks: 70

- Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B**)
 2. Answer **ALL** the question in **Part-A**
 3. Answer any **FOUR** Questions from **Part-B**

PART -A (14 Marks)

1. a) Find the rank of the matrix: $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{pmatrix}$. [2M]
- b) Find the Eigen values of A^3 where $A = \begin{pmatrix} 8 & -4 \\ 2 & 2 \end{pmatrix}$. [2M]
- c) Evaluate the double integral $\int_0^1 \int_x^{\sqrt{x}} xy \, dy \, dx$. [2M]
- d) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. [2M]
- e) Determine curl of $(xyz^2 i + yzx^2 j + zxy^2 k)$ at the point $(1, 2, 3)$. [2M]
- f) State the Green's theorem in plane. [2M]
- g) Write the real symmetric matrix associated with the quadratic form $x_1^2 + 4x_2^2 + x_3^2 - 4x_1x_2 + 2x_3x_1 - 4x_2x_3$. [2M]

PART -B (56 Marks)

2. a) Find P and Q such that the normal form of $A = \begin{pmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ is P A Q. [7M]
 Hence find the rank of A.
- b) Set up the Gauss-Jacobi iteration scheme to solve the system of equations $3x + 2y = 5$, $2x + 3y - z = 4$, $-y + 2z = 1$. Show that the iteration scheme converges. [7M]
3. a) Find a matrix P which diagonalizes the matrix $A = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$. Verify that $P^{-1}AP = D$. [7M]
- b) Reduce the quadratic form $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_2x_3 + 2x_1x_3 - 2x_1x_2$ to canonical form by orthogonal transformation. [7M]



4. a) Trace the following curve: $y^2(a + x) = x^2(b - x)$. [4M]
- b) Evaluate the double integral $I = \iint_R xy \, dx \, dy$, where R is the region bounded by the x-axis, the line $y = 2x$ and the parabola $y = x^2/4a$. [5M]
- c) Compute the volume of a sphere of radius r. [5M]
5. a) Show that $\beta(m, n) = \frac{\Gamma(m) \Gamma(n)}{\Gamma(m+n)}$. [7M]
- b) Evaluate i) $\int_0^{\infty} 2^{-9x^2} \, dx$ ii) $\int_0^1 \frac{dx}{\sqrt{-\log x}}$ using the Gamma function. [7M]
6. a) What is the directional derivative of $f(x, y, z) = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of the normal to the surface $x \ln z - y^2 = -4$ at $(-1, 2, 1)$ [7M]
- b) Prove that $(y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ is both Solenoidal and irrotational. [7M]
7. a) Use Green's theorem to evaluate the line integral $\oint_C [Mdx + Ndy]$ when $Mdx + Ndy$ equals to: $-y^3 dx + x^3 dy$ where C : circle $x^2 + y^2 = 1$. [7M]
- b) Using divergence theorem, evaluate the surface integral: $\iiint_S (x^3 dy \, dz + x^2 y \, dz \, dx + x^2 z \, dx \, dy)$ where closed surface consisting of the circular cylinder $x^2 + y^2 = a^2$, $(0 \leq z \leq b)$ and the circular disks $z = 0$ and $z = b$. [7M]

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